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WADT'22, Aveiro
June 29, 2022

Formal Methods for Quantum Fault-Tolerant Programs

A survey

Outline

Context

Problem

Current research

Future work

Context

Problem

Most Formal Methods for Quantum Computing (FMQC) consider ideal devices.

However, **noise** is everywhere,

and **Faulty Quantum Circuits** are here to stay!

Current Research

Quantum Error Correction and Software approaches

Quantum Error Correction (QEC)

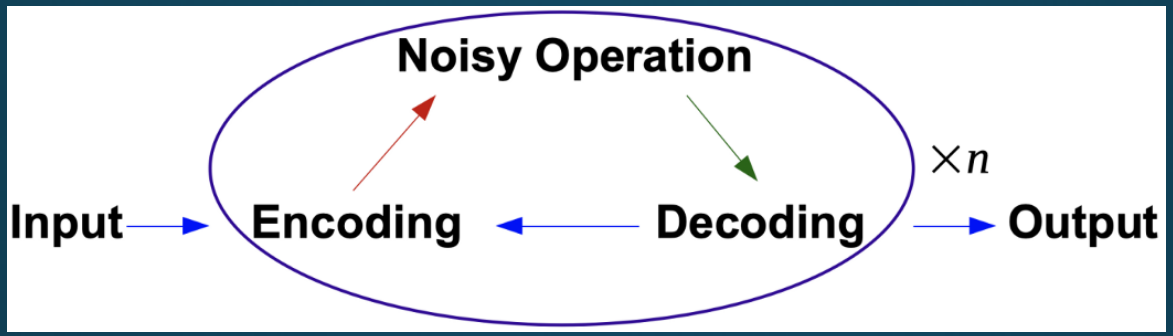

Figure: [Sandesara, 2021]

Quantum Error Correction (QEC)

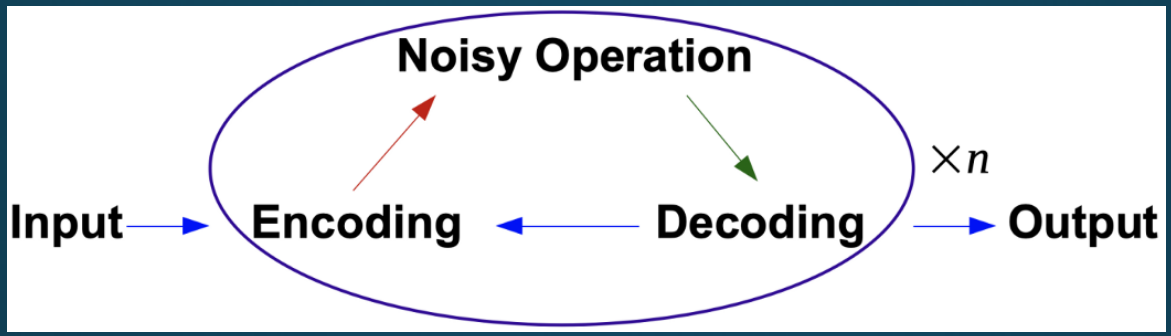

Figure: [Sandesara, 2021]

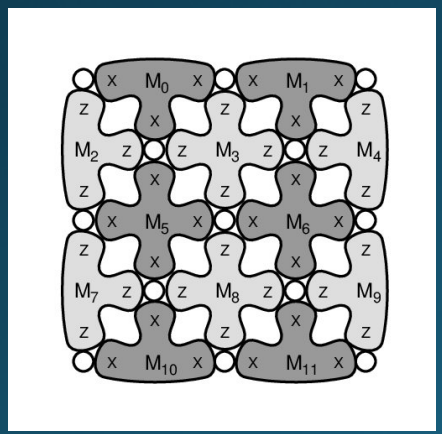

Figure: Surface code. [Fowler, 2012]

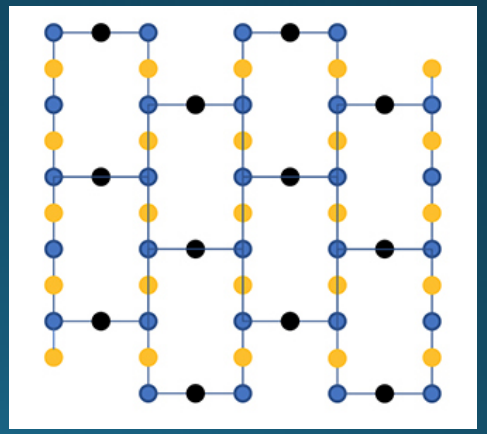

Figure: Heavy Hex lattice [Zhu and Cross, 2020]

What Do FMQC Verify? I

Functional Verification

- Can be done at different abstraction levels.

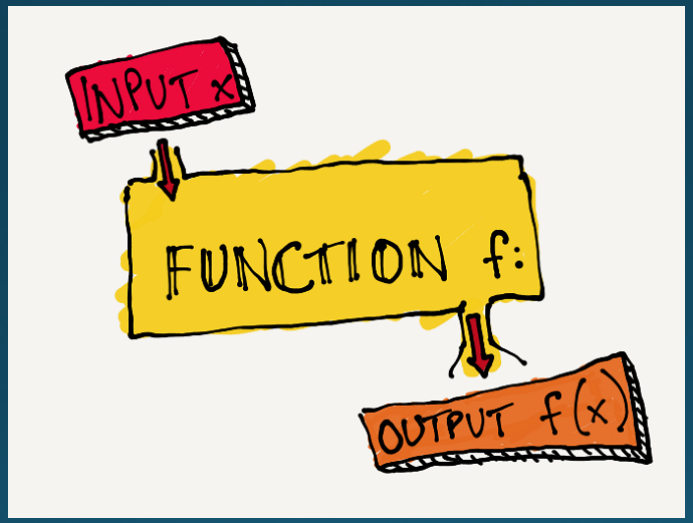

Figure: source

What Do FMQC Verify? II

Complexity Constrains

- Circuit metrics such as circuit depth and volume.

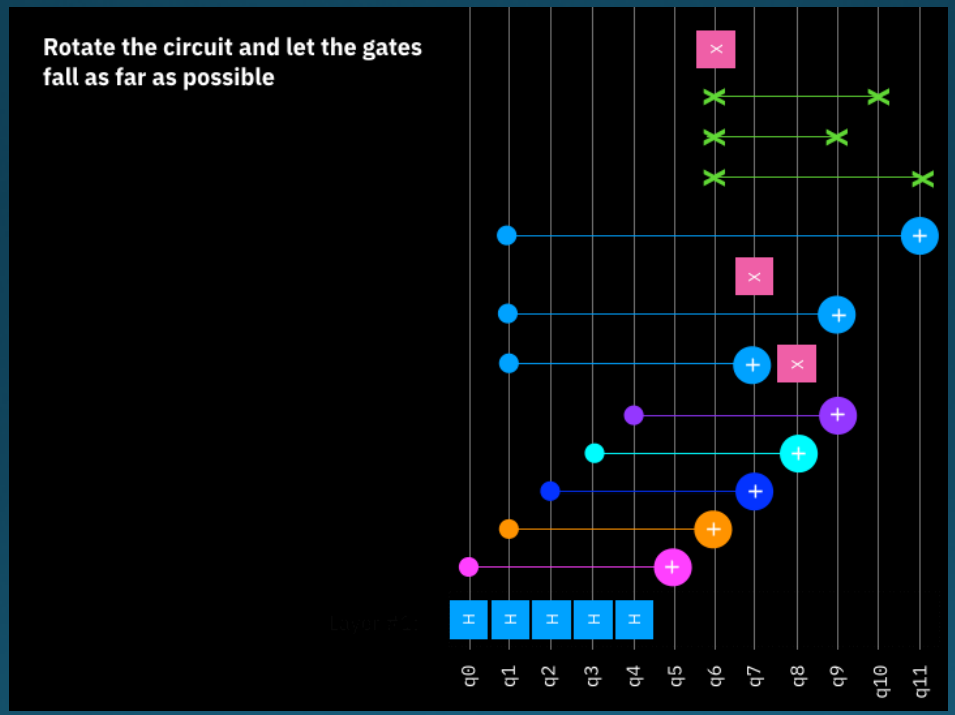

Figure: Circuit depth from Qiskit.

What Do FMQC Verify? III

Structural Constrains

- Target architecture and physical laws.
- Regulated by the language design or left for user's scrutiny.
- Can be syntactic or semantic.

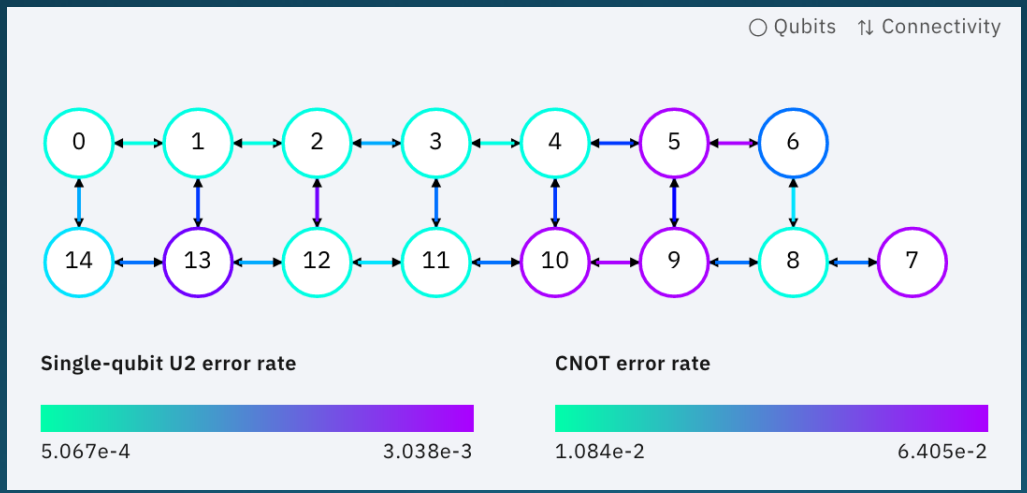

Figure: From IBM Q.

What Do FMQC Verify? IV

Circuit Equivalence

- Equivalence between quantum circuits implemented in quantum devices.

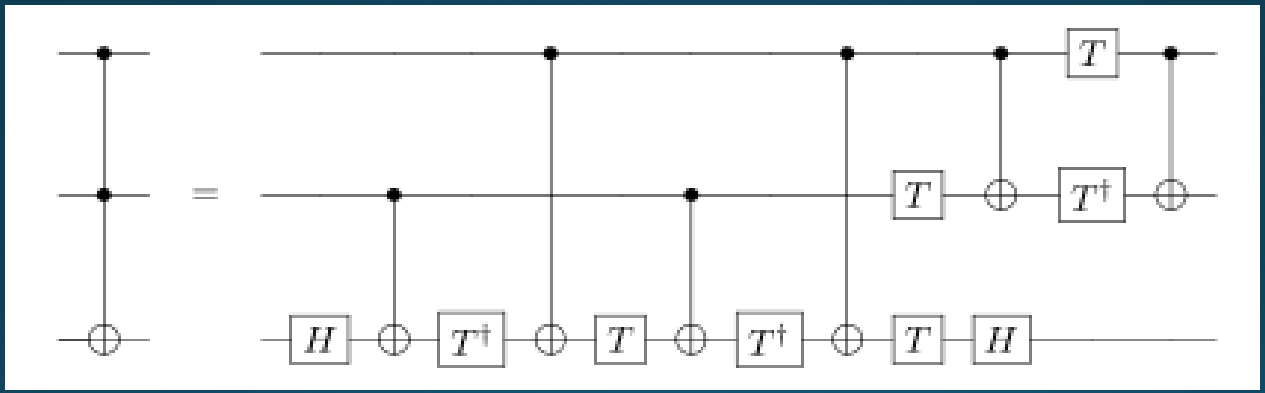

Figure: From Toffoli gate from Wikipedia.

Analyse to Quantum Error and Hardware Restriction I

Verifying quantum programs in the presence of errors [Rand et al., 2019]

- Proposes to aggregate errors in the circuit wire.
- Unsuitable to reason systems with error mitigation.

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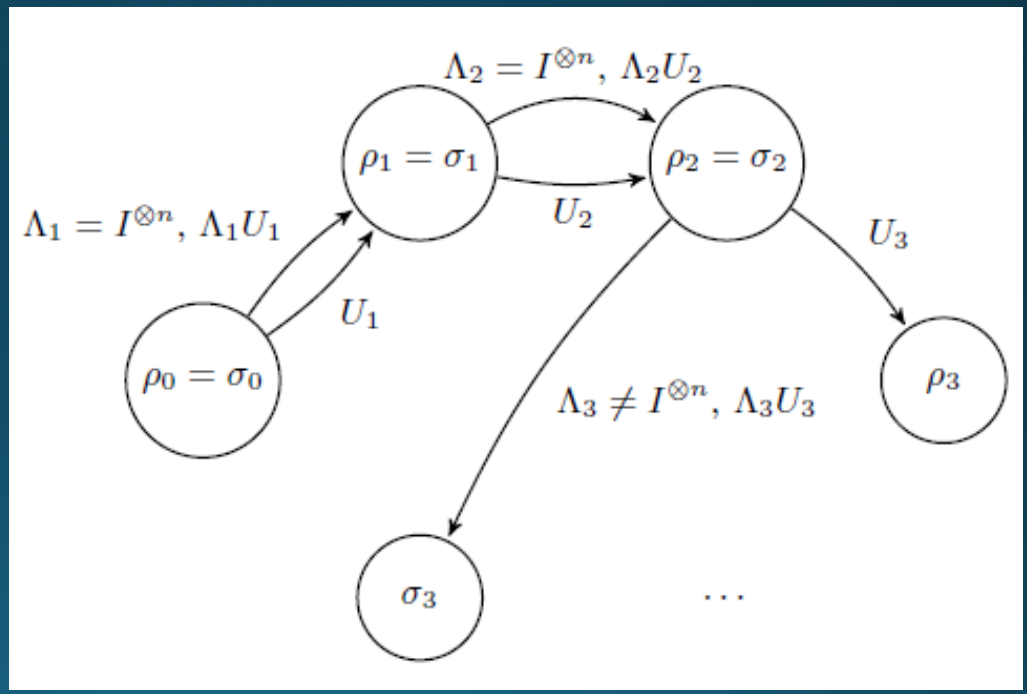
Example:

Ideal Quantum Gate: $Qubit -> Qubit$
Faulty Quantum Gate: $(Qubit, n) -> (Qubit, n+1)$

Analyse to Quantum Error and Hardware Restriction II

Combine Markov Chains to reason about error accumulation [Ma and Sanders, 2019]

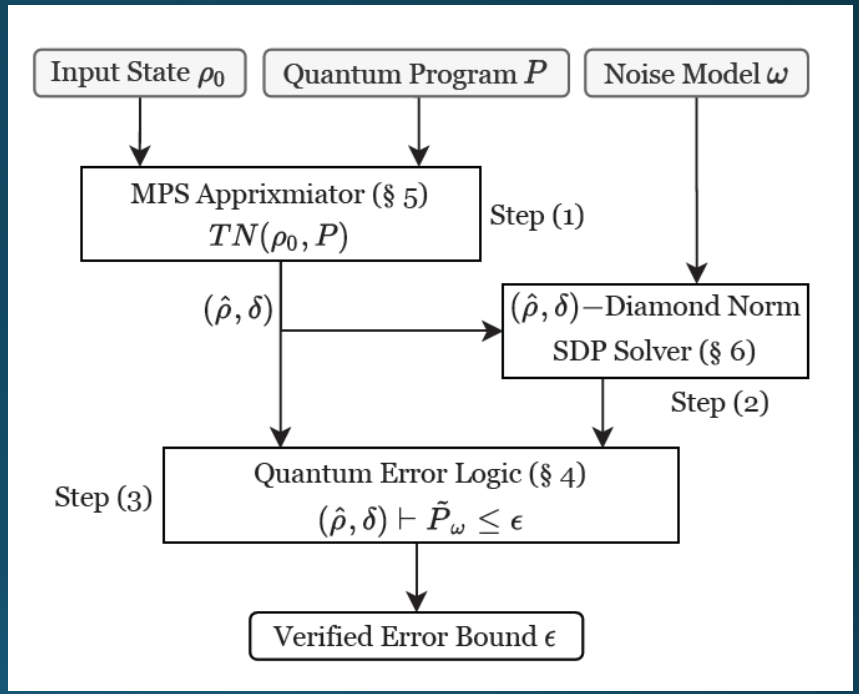
- How many quantum gates can be applied before many errors accumulate, with high probability?
- How can one lower error accumulation rates, in existing circuits, by using optimization techniques?



Analyse to Quantum Error and Hardware Restriction III

Quantum robustness and Gleipnir

- [Hung et al., 2019] extends quantum while languages with semantics that account for errors and define logic for reasoning about erroneous executions.

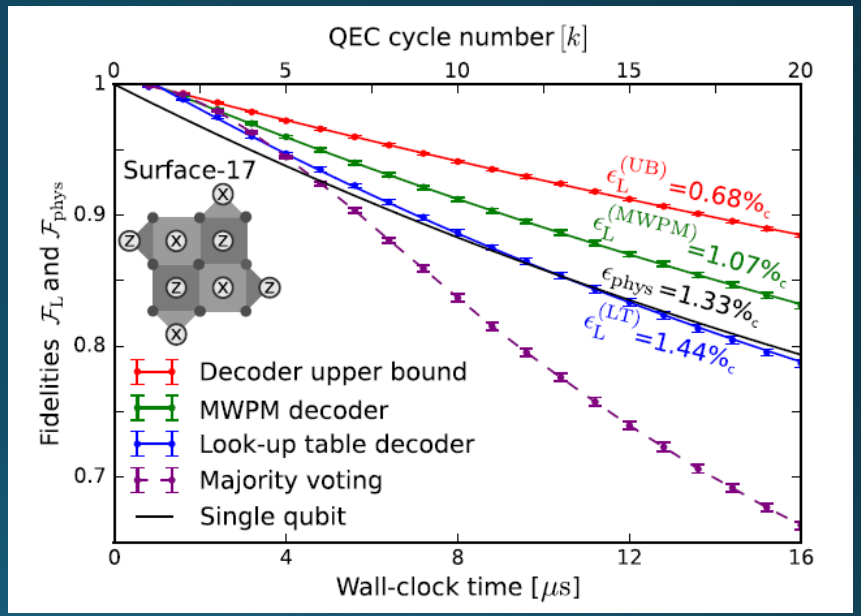


- [Tao et al., 2021] addresses practical questions left by [Hung et al., 2019].

Analyse to Quantum Error and Hardware Restriction IV

Density matrix simulation [Baek et al., 2019, O'Brien et al., 2017]

- To better understand the benefits of QEC, some works apply density matrix simulation.
- These studies can be used to optimize QEC schemes.



Analyse to Quantum Error and Hardware Restriction V

Verification of the compilation process [Meuli et al., 2020]

- Error may decrease by using more quantum resources.
- Allows for automatically selecting accuracy parameters while reducing quantum resource requirements, or vice-versa.

Future work

Considerations for future works

- For short term applications, most FMQC tools are insufficient.
- It is essential to focus on the limitations of quantum devices available.
  - Connectivity between qubits
  - Error Rates for each qubit
  - Gate fidelity
- Assessing circuit redundancy, robustness, or mitigation ability should also be a priority.

Concrete ideas

- Access if one can translate classical works on faulty hardware to the quantum paradigm.
- Extend quantum frameworks to consider faulty hardware (e.g. [Kissinger and van de Wetering, 2020])
- Studies on Abstract Interpretation (e.g. [Yu and Palsberg, 2021]) apply reduced density matrices to verify assertions. Similar applications may be a solution to the limited scalability of the standard density matrix simulation.

Questions

Funding: FCT/INL (SFRH/BD/151109/2021) and INESCTEC (LA/P/0063/2020)

References I

Baek, C., Ostuka, T., Tarucha, S., and Choi, B.-S. (2019).
Density matrix simulation of quantum error correction codes for near-term quantum devices.
Quantum Sci. Technol., 5(1):015002.

Fowler, A. (2012).
Proof of Finite Surface Code Threshold for Matching.
Phys. Rev. Lett., 109:180502.

Hung, S. H., Hietala, K., Zhu, S., Ying, M., Hicks, M., and Wu, X. (2019).
Quantitative robustness analysis of quantum programs.
Proc. ACM Program. Lang., 3(POPL):1–29.

References II

Kissinger, A. and van de Wetering, J. (2020).
PyZX: Large Scale Automated Diagrammatic Reasoning.
In Coecke, B. and Leifer, M., editors, Proceedings 16th International Conference on *Quantum Physics and Logic*, Chapman University, Orange, CA, USA., 10-14 June 2019, volume 318 of *Electronic Proceedings in Theoretical Computer Science*, pages 229–241. Open Publishing Association.

Ma, L. and Sanders, J. (2019).
Markov chains for error accumulation in quantum circuits.
arXiv, page ArXiv:1909.04432v1.

Meuli, G., Soeken, M., Roetteler, M., and Häner, T. (2020).
Enabling Accuracy-Aware Quantum Compilers using Symbolic Resource Estimation.
arXiv.

References III

O'Brien, T. E., Tarasinski, B., and DiCarlo, L. (2017).
Density-matrix simulation of small surface codes under current and projected experimental noise.
npj Quantum Inf., 3(39):1–8.

Rand, R., Hietala, K., and Hicks, M. (2019).
Formal Verification vs. Quantum Uncertainty.
In Lerner, B. S., Bodík, R., and Krishnamurthi, S., editors, *3rd Summit on Advances in Programming Languages (SNAPL 2019)*, volume 136 of *Leibniz International Proceedings in Informatics (LIPIcs)*, pages 12:1–12:11, Dagstuhl, Germany. Schloss Dagstuhl–Leibniz-Zentrum fuer Informatik.

Sandesara, H. (2021).
Quantum Error Correction using Qiskit - The Startup - Medium.
Medium.

References IV

Tao, R., Shi, Y., Yao, J., Hui, J., Chong, F. T., and Gu, R. (2021).
Gleipnir: toward practical error analysis for Quantum programs.
In *PLDI 2021: Proceedings of the 42nd ACM SIGPLAN International Conference on Programming Language Design and Implementation*, pages 48–64. Association for Computing Machinery, New York, NY, USA.

Yu, N. and Palsberg, J. (2021).
Quantum abstract interpretation.
In *PLDI 2021: Proceedings of the 42nd ACM SIGPLAN International Conference on Programming Language Design and Implementation*, pages 542–558. Association for Computing Machinery, New York, NY, USA.

References V

Zhu, G. and Cross, A. (2020).
Hardware-aware approach for fault-tolerant quantum computation | IBM Research Blog.
[Online; accessed 22. Jun. 2022].